Find the limit algebraically and verify numerically or graphically.

① $\lim_{x\to 0} \frac{x}{\sqrt{9+x}-3} \cdot \frac{\sqrt{9+x}+3}{\sqrt{9+x}+3} = \frac{\cancel{x}(\sqrt{9+x}+3)}{\cancel{9}+\cancel{x}-\cancel{9}} = \sqrt{9+x}+3$

$\lim_{x\to 0} \sqrt{9+x}+3 = \boxed{6}$

② $\lim_{x\to 0} \frac{\sin(3x)}{\sin(5x)}$

$\frac{5x}{5x} \; \frac{1}{\sin 5x} \cdot \frac{\sin 3x}{1}$

$\frac{5x}{\sin 5x} \cdot \frac{\sin 3x}{5x} \rightarrow \frac{\sin 3x}{5x} \div \frac{\sin 5x}{5x}$

$\frac{\sin 3x}{5x} \Rightarrow \frac{\sin 3x}{3x} \cdot \frac{3}{5}$

take $\lim_{x\to 0}$

$\frac{3}{5} \cdot \frac{\sin 3x}{3x} \div \frac{\sin 5x}{5x}$

$\downarrow \quad \downarrow \quad \downarrow$

$\frac{3}{5} \cdot 1 \div 1 = \frac{3}{5}$

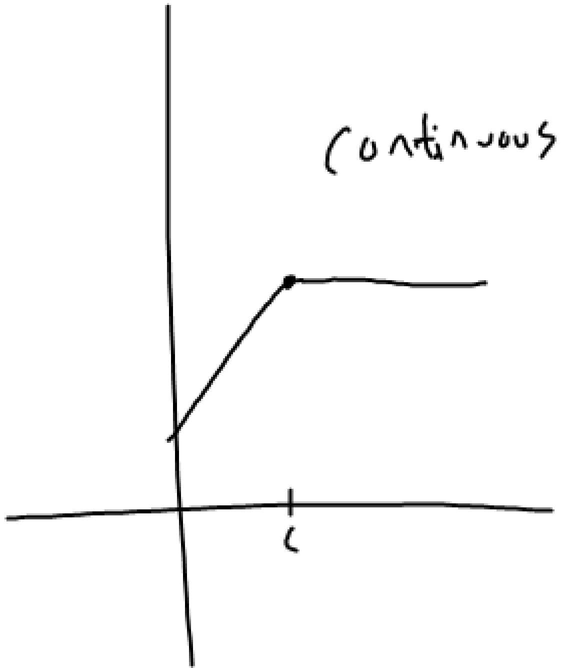
③ $\lim_{x\to 2} \frac{x^2-x-2}{x-2}$

$\frac{\cancel{(x-2)}(x+1)}{\cancel{(x-2)}} = \boxed{3}$

Continuity

interior point

1. $\lim_{x \to c} f(x)$ exists
2. $f(c)$ exists
3. $\lim_{x \to c} f(x) = f(c)$

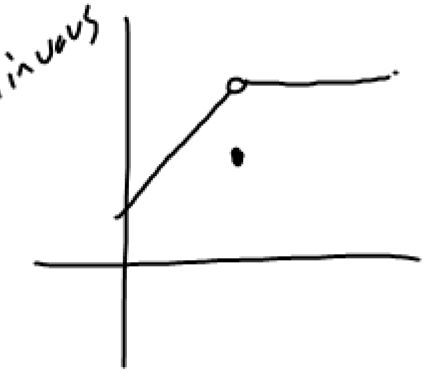


vs.

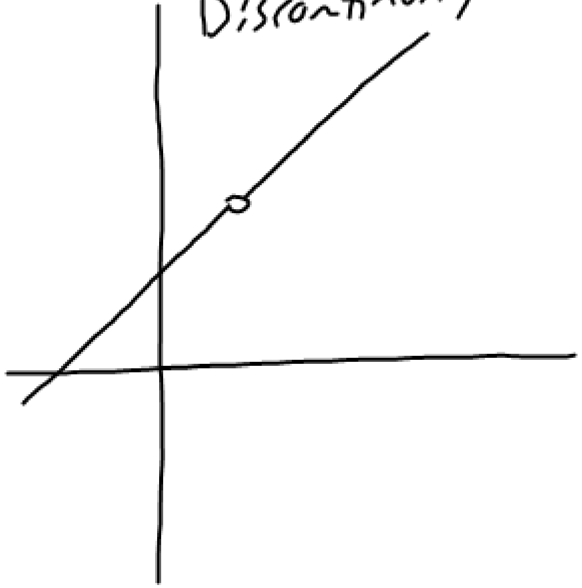
end point

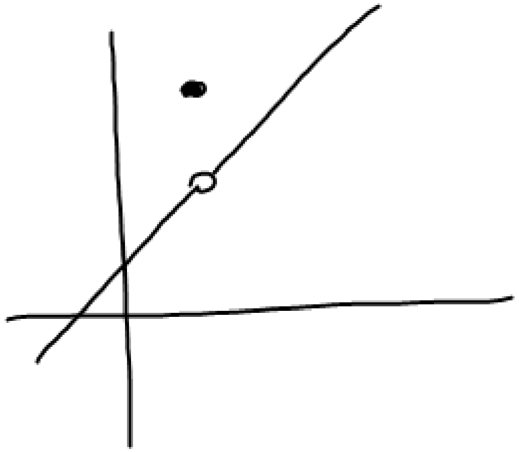
$\lim_{x \to a^+} f(x) = f(a)$ — left endpoint

$\lim_{x \to b^-} f(x) = f(b)$ - right endpoint

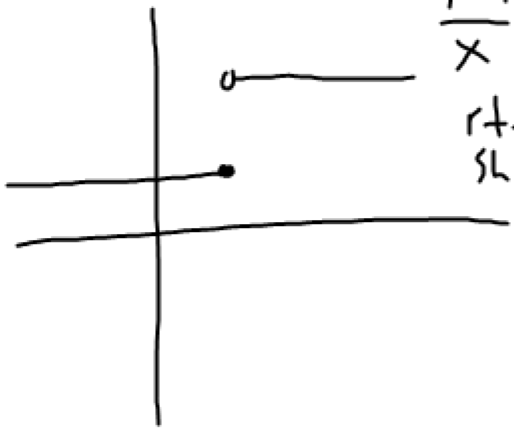


Removable Discontinuity





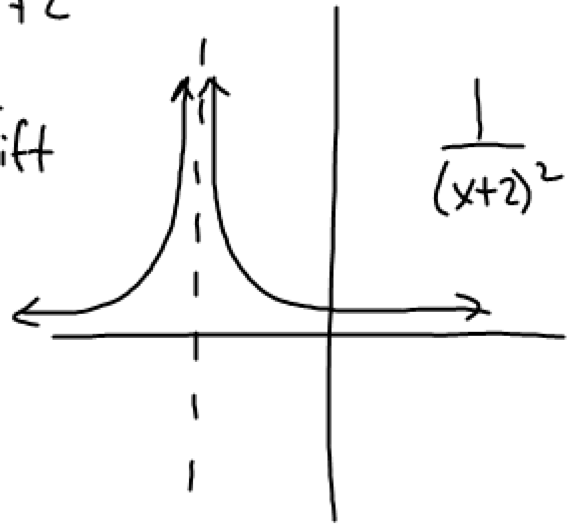
Jump Discontinuity



$\frac{|x|}{x}+2$

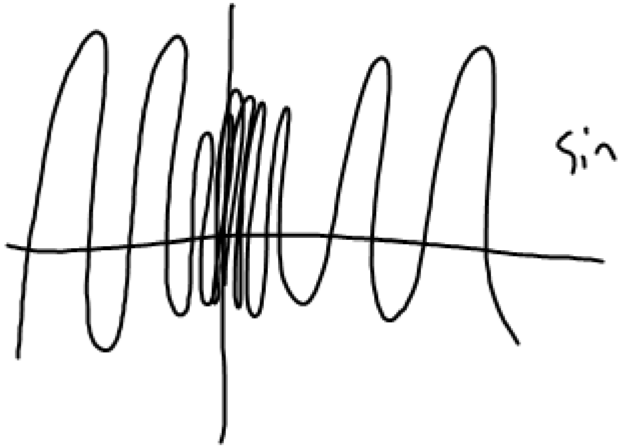
rt. shift

infinite discontinuity



$\frac{1}{(x+2)^2}$

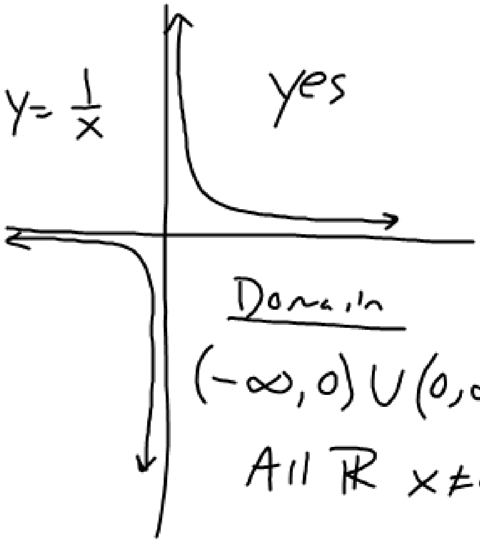
Oscillating discontinuity



$\sin\frac{1}{x}$

Continuous function is continuous at every point of its domain

$y = \frac{1}{x}$



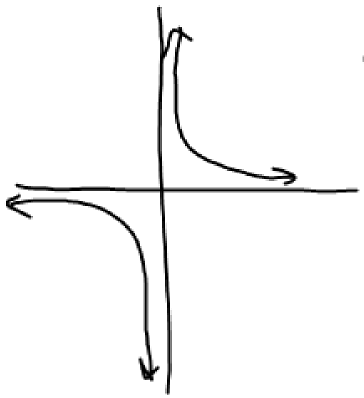
yes

Domain

$(-\infty, 0) \cup (0, \infty)$

All $\mathbb{R}$ $x \neq 0$

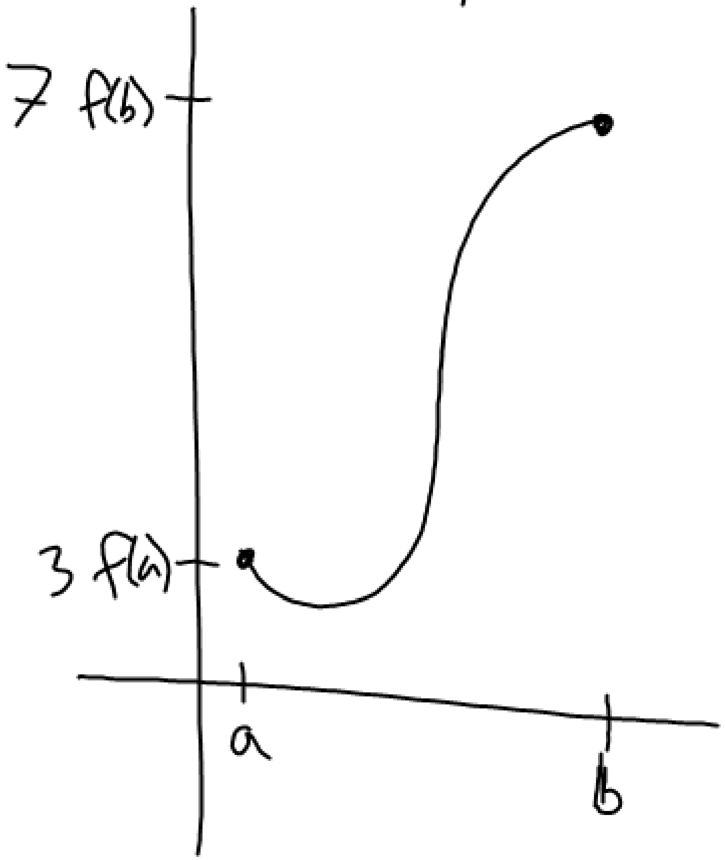
Continuous over an interval → every point must be continuous in that interval



Continuous over the interval $[-1, 1]$ = NO!

Intermediate Value Theorem

$$\sqrt[3]{\log x + \frac{1}{x}}$$



$\sqrt[3]{x}$ is continuous

$\log x$ is continuous

$\frac{1}{x}$ is continu

Sect. 2.3

#1-9 (odd), 11-14, 18-51 (5), 52, 53, 56, 58

$\log_y X + \log_x Y = \frac{10}{3}$ $\quad XY = 144$

$3(\log_x Y)^2 - 10(\log_x Y) + 3 = 0$ $\quad \frac{\log_x X}{\log_x Y}$ $\quad 3\log_x Y$

$\frac{1}{\log_x Y} + \log_x Y = \frac{10}{3}$ $\quad 3(\log_x Y)^2 + 3(\log_x X) = 10\log_x Y$

$\frac{10 \pm \sqrt{(-10)^2 - 4(3)(3)}}{2 \cdot 3}$ $\quad \frac{18}{6}$ $\quad \frac{2}{6}$ $\quad (\log_x Y - 3)\left(\log_x Y - \frac{1}{3}\right) = 0$

$3 \quad \frac{1}{3}$ $\quad \log_x Y - 3 = 0$

$XY = 144$

$X = \frac{144}{Y}$ $\quad \log_{\left(\frac{144}{Y}\right)} Y = 3$

$\left(\frac{144}{Y}\right)^3 = Y$ $\quad \frac{1}{2}(X + Y) = 22.52$

$144^3 = Y^4$

$(144^3)^{.25} = Y$ $\quad 144^{.75} = Y$

$Y = 41.569$ $\quad X = 3.4641$

$$\sum_{i=1}^{2010} f(x) \qquad f(x) = \begin{cases} \log_8 n & \text{if rational} \\ 0 & \text{otherwise} \end{cases}$$

$$\underset{0}{f(1)} + \underset{\frac{1}{3}}{f(2)} + \underset{0}{f(3)} + \ldots + \underset{0}{f(2009)} + \underset{0}{f(2010)}$$

$f(2) = \frac{1}{3}$

$f(4) = \frac{2}{3}$

$f(8) = 1$

$f(16) = 4/3$

$f(32) = 5/3$

$2, 4, 8, 16, 32, 64, 128, 256, 512, 1024,$ ~~2048~~

$\frac{1}{3}\ \ \frac{2}{3}\ \ 3/3\ \ 4/3$

$= \frac{55}{3}$

$$\log_2(\log_2(\log_2 x)) = 2$$

$$2^{(\cancel{\log_2}(\log_2(\log_2 x))} = 2^{(2)}$$

$$(\log_2(\log_2 x)) = 4$$

$$\cancel{2}^{(\cancel{\log_2}(\log_2 x))} = 2^{4}$$

$$\log_2 x = 16$$

$$\cancel{2}^{\cancel{\log_2} x} = 2^{16}$$

$$x = 65536$$

1 2 3 4 5

5 digits

(6) $\log_{2010}(\log_{2009}(\log_{2008}(\log_{2007}(x))))$

must be > 0

must be > 1 so $\log_{2007}(x) > 2008$

$x > 2007^{2008}$

$\log_{2010}?$ to work

? has > 0

$\log_{2009}? > 1$

$\log_{2008}? = 1$

$\log_{2007} x = 2008$

$x > 2007^{2008}$

$\{x \mid x > c\}$ = what is the smallest x

(c)